Volatility Estimates of the Short Term Interest Rate with an Application to German Data

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Abstract

This paper proposes a procedure for testing alternative specifications of the short term interest rate's dynamics which takes into account that according to some restrictions the interest rate is non-stationary, i.e. the traditional test statistic has a non-standard distribution. Moreover, we do not take the specification of the mean equation as given by the theory but rather base the decision of the lag structure on a robust Lagrange Multiplier test. In contrast to U.S. data we find that the volatility depends on either the interest rate level or information shocks but not on both. Finally, we propose to describe the short term interest rate's dynamics by means of an AR(1) model with stochastic volatility.

JEL Classification: C2, E4, G1

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1 Introduction

The specification of the stochastic differential equation of the instantaneous rate of interest and its volatility in particular is fundamental for pricing contingent claims or bonds. However, the empirical literature on term structure models used to lag behind the available theory. In recent years, though, an impressive amount of articles has emerged aiming at the correct specification of the short term interest rate dynamics. This especially holds with respect to those term structure models which JARROW (1995) calls *zero curve arbitrage models*, i.e. term structure models which take the stochastic differential equation of the instantaneous risk free rate of interest and a few bond prices as given in order to evaluate the remaining default free zero coupon bond prices. The other class of models is called *contingent claim valuation models*. Within these, no measurement error in calculating option prices emerges because in addition to the stochastic differential equation of futures prices or the instantaneous risk free rate of interest, the entire zero coupon bond price curve is taken as given.

This paper focuses on zero curve arbitrage models. CHAN/KAROLYI/LONGSTAFF/SANDERS (1992), CKLS, compare a number of zero curve arbitrage models by using an observable short term interest rate as an approximation for the theoretical instantaneous rate of interest and by using a crude discretisation for the continuous time models. A much cited result of their study is the point estimate for the levels effect parameter γ of 1.5 (see Table 1 for a definition) which implies non-stationarity for the interest rate process, thereby violating the ergodicity assumption of the applied GMM estimator (BLISS/SMITH (1997)).[1] The CKLS analysis has been extended in

[1] The non-stationarity of the interest rate process for $\gamma > 1$ is pointed out in DAHLQUIST (1996) and also mentioned in GOURIÉROUX/MONFORT (1996) or BROZE/SCAILLET/ZAKOÏAN (1995).

various ways. Brenner/Harjes/Kroner (1996), for instance, show that according to their data the volatility function incorporates both, a levels effect and autoregressive conditional heteroskedasticity (ARCH). For monthly data, three proposed models deliver point estimates of γ between 0.5 and 1.44. Bliss/Smith (1997) argue that the results derived in Chan et al. (1992) are invalid due to model misspecification. The monetary experiment by the Federal Reserve Board from October 1979 to September 1982 led to a structural break in the data generating process which is not accounted for in the CKLS analysis.

This paper also takes the CKLS model as a starting point for analysing German short term interest rates.[2] The crude discretisation of the continuous time models is used although there exist estimation techniques which try to eliminate the discretisation bias (e.g. Duffie/Singleton (1993), or Gallant/Tauchen (1996)). The justification is twofold: On the one hand, the continuous time models need in any case be applied to discrete data. The practitioner would probably like to know which of the zero curve arbitrage models fares best in this context. On the other hand, however, the efficient method of moments developed in Gallant/Tauchen (1996) as well as the indirect inference estimator of Gouriéroux/Monfort/Renault (1993) demands an auxiliary parametric model as a starting point for an estimate of the conditional density for the interest rate series. In this sense, this paper might be a preliminary study for either of the two methods.

The theory typically prescribes an AR(1) process for the short term interest rate. From an econometricians point of view, this might not be sufficient. This is why we employ the robust Lagrange Multiplier test (RB-LM test) developed in Wooldridge (1991) for the purpose of identifying a correct lag structure in the mean equation. Whereas the classical LM test is misspecified

[2] To our knowledge there does not exist a study in this context with German data.

in the presence of ARCH effects in the residuals, the RB-LM test is not. According to our results the latter does not reject the AR(1) model.

We propose a consistent method for testing the restrictions of alternative zero curve arbitrage models. The test statistic used in CHAN ET AL. (1992) does not have a standard distribution if the restrictions imply non-stationarity of the data generating process. In contrast to ANDERSEN/LUND (1997) and BRENNER/HARJES/KRONER (1996), we find for weekly data of the Eurocurrency DM 3-Month rate that its volatility depends either on the interest rate level or on information shocks but not on both. The results do not indicate a structural break in the data generating process for the time of the monetary experiment of the Federal Reserve Board. After testing various one factor zero curve arbitrage models and econometric specifications we derive a parsimonious time continuous model with stochastic volatility for the short term interest rate. Accordingly, two factors serve as the building block for a term structure model of interest rates in Germany.

The remainder of this article is organised as follows. Section 2 discusses the single factor models as well as the data set to be studied and explains the econometric methodology to be employed. In Section 3 the empirical results are reported and a term structure model is derived. A summary and concluding remarks complete the paper.

2 Theory and Econometric Methodology

2.1 One Factor Zero Curve Arbitrage Models

This section deals with term structure models which assume that a single stochastic factor causes the evolution of the entire zero coupon bond price curve. I.e. all interest rates are perfectly correlated with one single state variable, the instantaneous risk free rate of interest approximated by an observable short term interest rate in practice. As in CKLS, the single-factor diffusion processes to be studied can be nested in the following stochastic differential equation for the instantaneous risk free rate of interest r:

$$dr = (a + b\,r)dt + \sigma r^{\gamma} dz \tag{1}$$

where dz denotes the standard Wiener process or Brownian motion ($dz = \epsilon\sqrt{t}$, $\epsilon \sim N(0,1)$), and σr^{γ} the instantaneous standard deviation of interest rate changes which is often referred to as 'volatility'. The dependence of the instantaneous standard deviation on r^{γ} is known as the 'levels effect'. Within the models covered here, dz is the single factor driving the evolution of the entire term structure. Table 1 reports the term structure models included in (1). The specifications were chosen because of analytical tractability and intuition. The VASICEK, CIR-SR, and BRENNAN/SCHWARTZ models assume 'mean reversion', i.e. the interest rate is pulled toward its long term mean by the rate $|b|$.[3] Obviously, this imposes stationarity on the data generating process. The approximate discrete-time analog of the continuous-time model in equation (1) is (CKLS model)

$$\begin{aligned} r_t - r_{t-1} &= \alpha + \beta r_{t-1} + u_t \\ E[u_t|F_{t-1}] &= 0, \quad E[u_t^2|F_{t-1}] = h_t \\ h_t &= \sigma^2 r_{t-1}^{2\gamma} \end{aligned} \tag{2}$$

where F_t denotes the information set at time t, and $\sigma^2 r_{t-1}^{2\gamma}$ the (conditional) variance of interest rate changes. The restrictions $\beta = 0$ as well as $\gamma = 1.5$ give a non-stationary data generating process (see e.g. DAHLQUIST (1996)). Restricting the parameters to these values leads to a test statistic with a non-standard distribution and consequently unknown critical values. Therefore

[3] This can clearly be seen if (1) is written as $dr = -b(-a/b - r)dt + \sigma r^{\gamma} dz$ (with $b < 0$) where $|a/b|$ is the long term mean of r.

Table 1: Single-Factor Term Structure Models

Alternative single-factor zero curve arbitrage models are nested in

$$dr = (a + b\,r)dt + \sigma r^{\gamma} dz$$

Model		Restrictions a	b	γ	σ
MERTON[a]	$dr = adt + \sigma dz$	—	0	0	—
GBM[b]	$dr = b\,rdt + \sigma rdz$	0	—	1	—
DOTHAN[c]	$dr = \sigma rdz$	0	0	1	—
VASICEK[d]	$dr = (a + b\,r)dt + \sigma dz$	—	—	0	—
CIR-SR[e]	$dr = (a + b\,r)dt + \sigma\sqrt{r}dz$	—	—	0.5	—
BSCH[f]	$dr = (a + b\,r)dt + \sigma rdz$	—	—	1	—
CIR-VR[g]	$dr = \sigma r^{1.5}dz$	0	0	1.5	—
CEV[h]	$dr = b\,rdt + \sigma r^{\gamma}dz$	0	—	—	—

[a]MERTON (1973).

[b]Geometric Brownian Motion as used in RENDLEMAN/BARTTER (1980).

[c]DOTHAN (1978).

[d]VASICEK (1977)

[e]The CIR square-root model (COX/INGERSOLL/ROSS (1985)).

[f]BRENNAN/SCHWARTZ (1980).

[g]The CIR variable rate model (COX/INGERSOLL/ROSS (1980)).

[h]Constant Elasticity of Variance model as discussed in COX (1975) and COX/ROSS (1976).

we propose to first employ stationarity tests. These in combination with volatility estimates can determine whether interest rates should be assumed to be mean reverting in linear parametric models. In case of stationarity mean reversion and $\gamma \leq 1$ follow whereas non-stationarity could be due to $\gamma > 1$ and/or a non-mean reverting data generating process. Only if γ is estimated to be smaller than one and the restriction $\gamma = 1$ is rejected, the test result of non-stationarity is unambiguous.[4]

As pointed out by BLISS/SMITH (1997), this model might be misspecified with regard to the probable change in the process during the late 1970s and early 1980s. As Figure 1 on page 12 suggests, both the level as well as the volatility appear elevated. Since this period coincides with the temporary monetary targeting experiment of the Federal Reserve Board it is to be concluded that the U.S. market strongly influenced German rates. Following BLISS/SMITH (1997), a dummy variable is introduced for this period:

$$\begin{aligned} & r_t - r_{t-1} = (\alpha + \delta_1 D_t) + (\beta + \delta_2 D_t) r_{t-1} + u_t \\ & E[u_t | F_{t-1}] = 0, \quad E[u_t^2 | F_{t-1}] = h_t \\ & h_t = (\sigma^2 + \delta_3 D_t) r_{t-1}^{2(\gamma + \delta_4 D_t)} \end{aligned} \tag{3}$$

where

$$D_t = \begin{cases} 1 & \text{for}\, t \in (\text{Oct. 1979 until Sept. 1982}) \\ 0 & \text{other} \end{cases}$$

Moreover, BRENNER ET AL. (1996) show that for U.S. data the volatility of the short term interest rate needs to be modeled as a function of both the level as well as information shocks. The former is included in (3) because the lagged interest rate level directly affects its conditional variance.

[4] We restrict ourselves to the case where r follows a finite AR process. BACKUS/ZIN (1993) propose a one factor term structure model with fractional integration where r is non-stationary and yet mean reverting.

Information shocks are introduced into the volatility function by specifying an ARCH model.[5] We follow BRENNER ET AL. (1996) and use their AR(1)-GARCH(1,1)-X model which is an extension of the GARCH model as developed in BOLLERSLEV (1986):[6]

$$\begin{aligned} r_t - r_{t-1} &= \alpha + \beta r_{t-1} + u_t \\ E[u_t|F_{t-1}] &= 0, \quad E[u_t^2|F_{t-1}] = h_t \\ h_t &= c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1} + c_3 r_{t-1}^{2\gamma} \end{aligned} \tag{4}$$

Alternatively, we adopt the EGARCH model (NELSON (1991)) because ANDERSEN/LUND (1997) show that it fits their interest rate data best. However, we modify it to get a specification (AR(1)-EGARCH(1,1)-X) which is comparable to the GARCH-X model:

$$\begin{aligned} r_t - r_{t-1} &= \alpha + \beta r_{t-1} + u_t \\ u_t &= \eta_t \sqrt{h_t}, \quad \eta_t \sim i.i.d.\, N(0,1) \\ \log(h_t) &= \omega_0 + \omega_1 g(\eta_{t-1}) + \omega_2(\log(h_{t-1})) + \omega_3 r_{t-1}^{2\gamma} \\ g(\eta_t) &= \Theta\eta_t + \vartheta[|\eta_t| - E[\eta_t]] \end{aligned} \tag{5}$$

Of course, the dummy variable as defined for the CKLS model would need to be added in the AR(1)-GARCH(1,1)-X as well as in the AR(1)-EGARCH(1,1)-X model. For tractability, these versions are not stated. In (5), the conditional variance is a function of the lagged absolute disturbance instead of the lagged squared disturbance. In addition, η_t enters directly the conditional variance equation which is known as a representation of the leverage effect. Negative shocks with respect to the expected bond prices are likely to be followed by an increased volatility whereas positive shocks

[5] LAMOUREUX/LASTRAPES (1990) argue that ARCH effects arise when information shocks are serially correlated.

[6] BOLLERSLEV/CHOU/KRONER (1992) and BERA/HIGGINS (1993), respectively, give an overview for ARCH models.

should lead to a reduced volatility. Due to the relationship between interest rates and bond prices one would expect the opposite to hold in the above model, i.e. $\omega_1\Theta$ is expected to be positive. The AR(1)-EGARCH(1,1)-X model allows the interest rate level to influence its conditional variance in two ways: Through the just described leverage effect and through the levels effect which is measured by the parameter ω_3.

Apart from the inclusion of asymmetry, this specification has two significant advantages. First, it ensures a positive correlation between the conditional variance and its lagged values, and lagged squared disturbances. Negative parameter estimates cannot a priori be ruled out in the GARCH-X model whereas theoretically it only is defined for positive parameter values. Second, for $c_1 + c_2 = 1$ in the GARCH specification the interest rate process would be covariance non-stationary and yet possibly strictly stationary. Within the EGARCH framework such a conflict does not arise (see e.g. ANDERSEN/LUND (1997) and the literature cited therein).

2.2 Econometric Methodology

We start the analysis with the CKLS model specification as given in equation (3) and continue with the GARCH and EGARCH models. In contrast to CKLS and BLISS/SMITH (1997), all models are estimated by Maximum Likelihood assuming normally distributed residuals. Alternatively, the Student-t distribution could have been employed but because of consistency considerations we prefer the former. This is the same approach as in ANDERSEN/LUND (1997). For properties of the Quasi Maximum Likelihood approach see also WEISS (1986) and BOLLERSLEV/WOOLDRIDGE (1992). The log-likelihood function to be maximised is

$$\log L(\boldsymbol{p}) = -\tfrac{1}{2}\left(\log(h_t) - \tfrac{u_t^2}{h_t}\right) \tag{6}$$

where $\boldsymbol{p}$ is the vector of parameters of the model to be estimated. Engle (1982) argues in his seminal paper that a consistent and efficient ML estimation demands a consistent initial estimate of the mean equation parameters. Therefore, we first estimate the mean equation by least squares and use its parameter estimates and residuals as initial values for the ML estimation. The log-likelihood function is maximised by the BFGS algorithm.

Apart from testing various volatility specifications, we test for the correct lag structure in the mean equation. In Brenner et al. (1996) as well as in Bliss/Smith (1997), misspecification tests are of major concern only insofar as they deal with the volatility function. This is especially surprising since Brenner et al. (1996) report Ljung-Box Q statistics which indicate the presence of serial correlation in all models. A justification may be that the theory prescribes an AR(1) process for the instantaneous risk free rate of interest. But in practice, this assumption does not necessarily hold with respect to an observable short rate (an exception is Andersen/Lund (1997): none of their two-factor models does exhibit serial correlation in the residuals of the mean equation). The argument in Engle (1982) gives a justification for neglecting serial correlation in the conditional mean for ARCH models with a block diagonal information matrix. Accordingly, conditional mean and conditional variance can be estimated independently without a loss of asymptotic efficiency. But this argument does not hold for asymmetric ARCH models such as the EGARCH specification.

Diebold (1986) points out that the Ljung-Box test for serial correlation is misspecified in the presence of ARCH effects because they invalidate the standard asymptotic distribution theory. Therefore, the robust LM test (RB-LM test) developed in Wooldridge (1991) is employed (Brenner et al. (1996) use this kind of test for diagnostics of the volatility function). The terminology refers to the fact that the test statistic is robust with regard to a possibly misspecified volatility function. The following paragraph briefly discusses the RB-LM test.[7]

The first step involves a standardisation of the estimated residuals ($\hat{u}_t$) which are to be tested for serial correlation:

$$\tilde{\mathbf{x}}_t = \mathbf{x}_t \left(\sqrt{\hat{h}_t}\right)^{-1}, \ \tilde{\hat{u}}_{t-i} = \hat{u}_{t-i} \left(\sqrt{\hat{h}_t}\right)^{-1}, \ i = 0, \ldots, k \tag{7}$$

where $\mathbf{x}_t$ denotes the vector of regressors used in the mean equation and k is the lag order which is to be used in the test for serial correlation. Next, the effect of the regressors on lagged residuals is eliminated by means of the following linear regressions

$$\tilde{\hat{u}}_{t-i} = \tilde{\mathbf{x}}_t' b + \tilde{\hat{u}}^*_{t-i} \quad i = 1, \ldots, k. \tag{8}$$

This would give the following test regression:

$$\tilde{\hat{u}}_t = \lambda_1 \tilde{\hat{u}}^*_{t-1} + \ldots + \lambda_k \tilde{\hat{u}}^*_{t-k} + v_t \tag{9}$$

Instead, WOOLDRIDGE (1991) proposes to multiply (9) by $\tilde{\hat{u}}_t$ and take the conditional expectation which gives

$$1 = \rho_1 \tilde{\hat{u}}^*_{t-1} \tilde{\hat{u}}_t + \ldots + \rho_k \tilde{\hat{u}}^*_{t-k} \tilde{\hat{u}}_t + w_t \tag{10}$$

where w_t denotes the expectation error. The test statistic is the number of observations (T) minus the sum of squared residuals (SSR) of (10) with $T - SSR \sim \mathcal{X}^2(k)$ under the null hypothesis. This test is called *robust* Lagrange Multiplier (RB-LM) test because the estimation of the covariance matrix of $\tilde{\hat{u}}^*_{t-i} \tilde{\hat{u}}_t$ is not affected by the specification of the function for h_t.

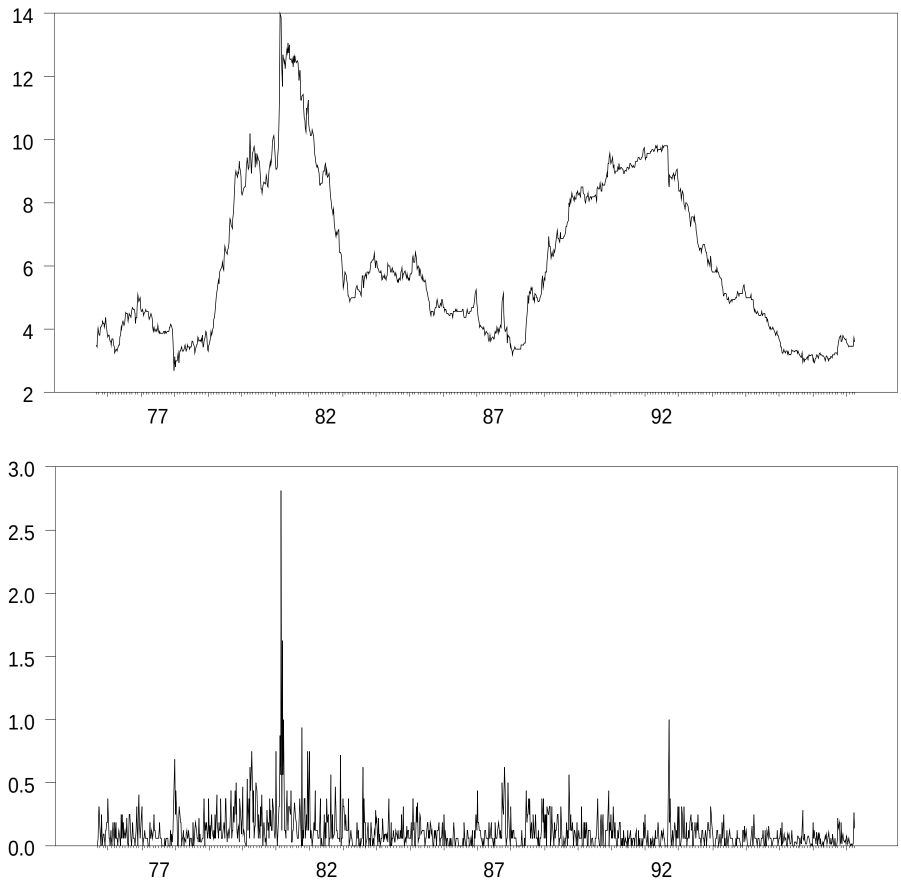
The Data

In this study, the Eurocurrency DM 3-Month rate (London market, $R_{tD}^{(3m)}$) with weekly observations, supplied by Datastream, is used. The data covers

[7] An application and description can also be found in DANKENBRING/MISSONG (1997).

the period February 1975 until the beginning of April in 1998, i.e. 1210 observations in total. With respect to U.S. data, DUFFEE (1996) argues, that instead of the 1-Month rate, the 3-Month rate is better suited as a proxy for the theoretical instantaneous risk free rate of interest. Weekly sampled data is likely to lead to a smaller discretisation bias than monthly data. Figure 1 shows the series as well as the absolute changes.

Figure 1: The Euro-DM 3-Month Rate and its Absolute Changes



In the case of finite AR processes the econometric concept of stationarity corresponds to the theoretical concept of mean reversion. Therefore stationarity tests are an important tool for detecting the correct model specification. For this purpose we employ the KPSS test, derived in KWIATKOWSKI/PHILLIPS/SCHMIDT/SHIN (1992), as well as the augmented

Dickey/Fuller (ADF) and the Phillips/Perron (PP) test. In contrast to the latter two, the first tests the null hypothesis of stationarity against the alternative of a unit root. The following paragraph briefly introduces the test. Since the data and interest rates in general do not show a deterministic time trend for a long enough sample period we restrict ourselves to the case of testing for level stationarity.

First, the variable z_t to be tested is regressed on an intercept and the corresponding residuals e_t are computed (i.e. $e_t = z_t - \bar{z}$, $t = 1, \ldots, T$). Next, the partial sum process of e_t, S_t, is defined as

$$S_t = \sum_{i=1}^{t} e_i \qquad t = 1, \ldots T. \tag{11}$$

The test statistic is

$$\eta = T^{-2} \sum_{t=1}^{T} S_t^2 / \sigma^2 \tag{12}$$

where σ^2 is the long run variance defined as

$$\sigma^2 = \lim_{T \to \infty} T^{-1} E[S_T^2]. \tag{13}$$

Of course, σ^2 is not observable. A consistent estimator denoted by $s^2(l)$ is constructed from the residuals e_t in the following way:

$$s^2(l) = \frac{1}{T} \sum_{t=1}^{T} e_t^2 + \frac{2}{T} \sum_{g=1}^{l} \left(1 - \frac{g}{l+1} \right) \sum_{t=g+1}^{T} e_t e_{t-g}. \tag{14}$$

Finally the estimated test statistic denoted by $\hat{\eta}$ is

$$\hat{\eta} = T^{-2} \sum_{t=1}^{T} S_t^2 / s^2(l). \tag{15}$$

Unfortunately, the test statistic is dependent on the choice of the lag truncation parameter l. For small values a considerable size distortion might arise due to significant autocorrelation in the residuals e_t. On the other hand, the power under the alternative decreases as l increases because $s^2(l)$ increases and consequently the test statistic decreases as l increases by construction. Kwiatkowski et al. (1992) argue that a good compromise between large size distortions and small power under the alternative is given for $l = 8$. However, Table 2 shows the test statistics for $l = 0, \ldots, 12$.

Table 2: KPSS Test for Stationarity

l^a	0	1	2	3	4	5	6
Test stat. for ${r_t}^b$	8.02	4.02	2.68	2.02	1.62	1.35	1.16
Test stat. for $\triangle {r_t}^c$	0.33	0.32	0.32	0.32	0.32	0.31	0.30
l	7	8	9	10	11	12	
Test stat. for r_t	1.01	0.91	0.82	0.75	0.69	0.63	
Test stat. for $\triangle r_t$	0.30	0.30	0.30	0.29	0.28	0.28	

[a] l denotes the lag truncation parameter of the long run variance estimator. The critical values derived in Kwiatkowski et al. (1992) for a significance level of 5% (1%) are 0.463 (0.739).

[b] This row gives the test statistics for r_t.

[c] This row shows the test statistics for $\triangle r_t = r_t - r_{t-1}$.

For $l = 0, \ldots, 10$ the null of stationarity is rejected at the 1% level, for $l = 11, 12$ at the 5% level whereas the null of difference stationarity can clearly not be rejected.

Table 3 gives the results of the more standard ADF and PP test for stationarity. First, the ADF test regression was run with a constant, i.e. under the hypothesis of a deterministic linear time trend in the level. This gives a test for trend stationarity. Since the intercept always turned out to be insignificant we also here report the test results only for the level stationarity case.

The tests imply that the Euro-DM 3-Month rate is a random variable of a data generating process which is integrated of order one.

Table 3: ADF and PP Test for Stationarity

	ADF test[a]		PP test	
lag[b]	$\triangle r_t$	r_t	$\triangle r_t$	r_t
1	-25.39	-1.39	-32.23	-1.37
2	-21.48	-1.27	-32.18	-1.36
3	-16.66	-1.19	-32.15	-1.34
4	-14.89	-1.33	-32.15	-1.35
5	-13.90	-1.36	-32.15	-1.37
6	-12.83	-1.34	-32.16	-1.38

[a]These rows show the ADF test statistics. Within this model without an intercept (i.e. the time series does not contain a deterministic time trend) the critical value of the 1% significance level for both tests is -2.57 (cf. Davidson/McKinnon (1992)).

[b]With respect to the ADF test, "lag" denotes the maximum lag order, with respect to the PP test the truncation parameter for the Bartlett window.

We conclude that the German short rate does not exhibit a deterministic time trend and is to be modeled as a variable of an integrated process of order 1.[8] Consequently, the short term interest rate does not mean revert in our framework unless the volatility function causes non-stationarity. Although

[8]Ball/Torous (1996) perform simulation studies which show that neglecting non-stationarity yields misleading results for zero curve arbitrage models. This holds independently from the estimation technique used.

this poses conceptual difficulties because the model cannot rule out negative values in the future, it means that linear parametric empirical analyses have to be carried out within the econometric framework for non-stationary data.[9] There simply are too few observations for which the process mean reverts. Nor does the series exhibit a deterministic time trend.

3 Model Estimations

First, the CKLS model with dummy variables for the period of the monetary experiment of the Federal Reserve Board as given in equation (3) is estimated, Table 4 shows the results. The RB-LM(1) test statistic amounts to 0.11 with a marginal significance level of 0.74. The autocorrelation function is depicted in Figure 2.[10] The latter indicates serial correlation to be present in the residuals whereas the RB-LM test does not. Therefore the estimations are carried out with lagged interest rate differences as well as without. The coefficients of interest hardly alter at all and the additional coefficients are insignificant. Also a test for joint significance, i.e. $H_0 : \phi_1 = \phi_2 = \phi_3 = 0$, does not allow for a rejection of the null hypothesis (the $\mathcal{X}^2(3)$ distributed test statistic is 2.658, with a marginal significance level of 0.447).

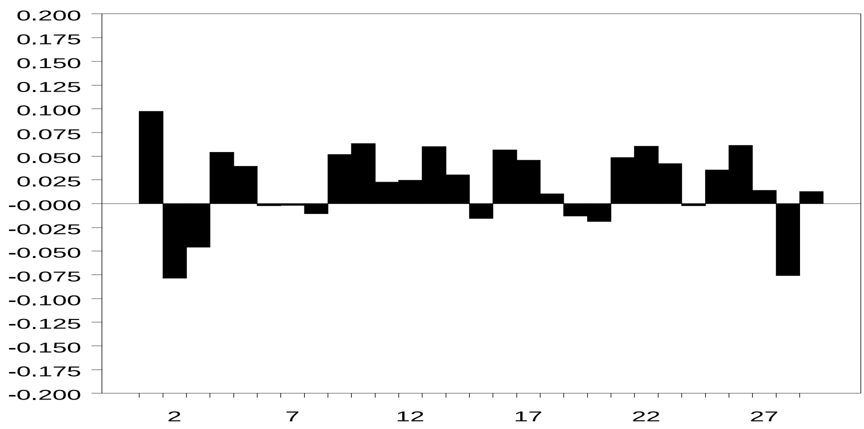
Also these estimates deliver a non-stationary data generating process. The levels effect parameter γ is equal to 0.12 and insignificant for both econometric models.[11] The dummy variables in the conditional variance equation are

[9] Also STOCK/WATSON (1993) mention these conceptual difficulties but nevertheless follow their test results and assume interest rates to be $I(1)$.

[10] According to the Bartlett approximation the null hypothesis of a negligible autocorrelation coefficient must be rejected if the estimated coefficient is greater than $2/\sqrt{T} = 2/\sqrt{1210} \approx 0.06$.

[11] The t-statistics of γ and its dummy parameter will only be valid if the true value is smaller than one. The estimates do not indicate a violation of this assumption.

Figure 2: Autocorrelation Function of CKLS Residuals without Lagged Interest Rate Differences



significant which implies that the model cannot explain the increased interest rate volatility during the early eighties. However, since the data plot exhibits two significant outliers for February/March 1981 the model is re-estimated without these observations.[12] Table 5 gives the results.

The parameter δ_3 remains significant but a joint test with $\delta_3 = \delta_4 = 0$ under the null hypothesis gives a $\mathcal{X}^2(2)$ distributed test statistic of 4.68 with a marginal significance level of 0.096. Therefore we conclude that there is no structural break in the data generating process. The one factor zero curve arbitrage models are to be tested within the traditional CKLS framework. Also the Hannan-Quinn information criterion favours the model without any dummies. As shown in Table 5 the CKLS model gives a point estimate of the levels effect parameter γ which is close to 0.5 and highly significant. The CIR-SR model assumes this particular value. Also if the outliers in March/February 1981 were eliminated from the sample does the CKLS model deliver this result (not given).[13]

[12] In Deutsche Bundesbank (1981) these values are explained by a temporary abandonment of its short term loan instrument called *Sonderlombard*.

[13] This is in contrast to previous estimates with monthly data. There, one single outlier

Table 4: Estimates of Levels Effect Model with Dummies

The model estimated with weekly data of the DM 3-Month rate is

$$r_t - r_{t-1} = (\alpha + \delta_1 D_t) + (\beta + \delta_2 D_t) r_{t-1}$$
$$+\phi_1 \triangle r_{t-1} + \phi_2 \triangle r_{t-2} + \phi_3 \triangle r_{t-3} + u_t$$
$$E[u_t|F_{t-1}] = 0,\ E[u_t^2|F_{t-1}] = h_t,\ h_t = (\sigma^2 + \delta_3 D_t) r_{t-1}^{2(\gamma+\delta_4 D_t)}$$
$$D_t = [1 \text{ for } t \in (5.10.1979 \text{ - } 24.9.1982), 0 \text{ other}].$$

	Model without lags		Model with lags	
α[a]	0.0132	(0.419)	0.0143	(0.460)
δ_1	0.2741	(0.940)	0.2774	(0.922)
β	-0.0028	(-0.491)	-0.0029	(-0.515)
δ_2	-0.0259	(-0.865)	-0.0263	(-0.853)
ϕ_1			0.0954	(1.469)
ϕ_2			-0.0389	(-0.605)
ϕ_3			-0.0014	(-0.021)
σ^2	0.0981	(2.533)	0.0979	(2.883)
δ_3	-0.0964	(-2.509)	-0.0963	(-2.767)
γ	0.1226	(1.079)	0.1227	(1.196)
δ_4	1.0310	(2.877)	1.0392	(4.234)
HQ[b]	-1.972		-1.964	
RB-LM(1) Test[c]	0.1101	(0.740)	1.396	(0.237)
RB-LM(11) Test	10.36	(0.499)	7.775	(0.733)

[a] t-values are in brackets.

[b] Hannan-Quinn information criterion.

[c] Marginal significance levels are in brackets.

Chan et al. (1992) chose this framework for testing the restrictions of al-

significantly influenced the results as in Bliss/Smith (1997).

Table 5: Estimates of Levels Effect Model

The model estimated with weekly data of the DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \delta_1 D_{2/81} + \delta_2 D_{3/81} + \beta r_{t-1} + u_t$$

$$E[u_t|F_{t-1}] = 0,\ E[u_t^2|F_{t-1}] = h_t,\ h_t = (\sigma^2 + \delta_3 D_t) r_{t-1}^{2(\gamma+\delta_4 D_t)}$$

$$D_t = [1 \text{ for } t \in (5.10.1979 \text{ - } 24.9.1982), 0 \text{ other}]$$

$$D_{2/81} = [1 \text{ for } t = \ 27.2.1981,\ 0 \text{ other}]$$

$$D_{3/81} = [1 \text{ for } t = \ 6.3.1981,\ 0 \text{ other}].$$

	CKLS model		Model without outl. and with Bliss/Smith dummies	
α^a	0.0146	(0.488)	0.0163	(0.641)
δ_1			2.8322	(5.235)
δ_2			-0.0964	(-0.140)
β	-0.0027	(-0.508)	-0.0032	(-0.688)
σ^2	0.0343	(3.308)	0.0980	(2.872)
δ_3			-0.0909	(-2.664)
γ	0.4671	(5.906)	0.1229	(1.158)
δ_4			0.6942	(1.483)
HQ^b	-2.129		-1.973	
RB-LM(1) Test^c	0.015	(0.903)	0.0278	(0.868)
RB-LM(11) Test	3.887	(0.973)	11.67	(0.389)

[a] t-values are in brackets.

[b] Hannan-Quinn information criterion.

[c] Marginal significance levels are in brackets.

ternative term structure models. However, the test statistic is not standardly distributed if a non-stationary DGP were assumed under the null. This is the case for $\beta = 0$ as well as $\gamma > 1$. We avoid such difficulties by first determining the characteristics of the mean equation and second analysing the properties of the conditional variance. The only testable restrictions are those on the levels effect parameter with $\gamma \leq 1$ under the null. Table 6 gives the results.

Table 6: Test of Alternative Zero Curve Arbitrage Models

The unrestricted econometric model is

$$\triangle r_t = \alpha + \beta r_{t-1} + u_t,\ E[u_t|F_{t-1}] = 0,\ E[u_t^2|F_{t-1}] = h_t,\ h_t = \sigma^2 r_{t-1}^{2\gamma}$$

Model	Testable Restrictions	Test statistic[a]
MERTON, VASICEK	$\gamma = 0$	35.14 (< 0.001)
CIR-SR	$\gamma = 1/2$	0.151 (0.728)
GBM, DOTHAN, BSCH	$\gamma = 1$	44.20 (<0.001)

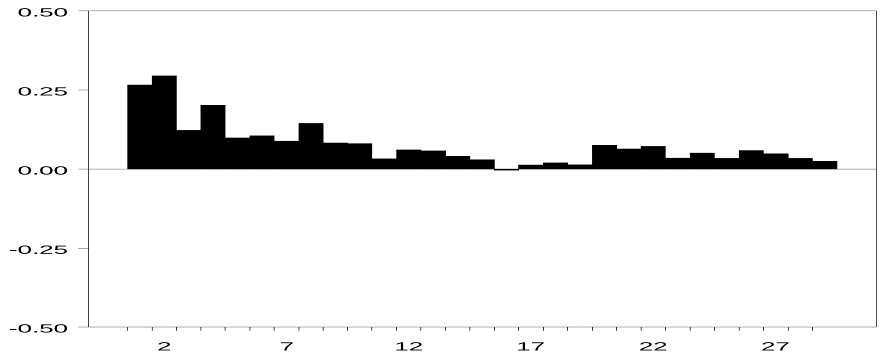
[a]The test statistic is distributed as $\mathcal{X}^2$ with one degree of freedom. Marginal significance levels are in brackets.

Not surprisingly, the only restriction which is not rejected is $\gamma = 0.5$. Consequently, the stationarity tests analysed in the last section taken together with these results propose a model without mean reversion. The zero curve arbitrage model suggested by the data thus far is

$$dr = \sigma r^{0.5} dz \tag{16}$$

i.e. a generalized Wiener process without a drift but with an instantaneous standard deviation which is dependent on the interest rate level. With respect to Table 1 the restrictions are $\alpha = 0, \beta = 0$ und $\gamma = 1/2$. However, the autocorrelation function of the absolute standardised residuals of the unrestricted CKLS model (i.e. the autocorrelation function of u_t/h_t with u_t and h_t as given in equation (2), Figure 3) suggests that the conditional variance time dependence is not adequately modeled. Accordingly, a GARCH specification is to be preferred.

Figure 3: Absolute CKLS Residuals Autocorrelation Function



First, the most general GARCH-X specification which includes dummies for the monetary experiment period is analysed. Table 7 shows the results. The model with an exact fit for the two outliers does not indicate a structural break. Consequently, the GARCH model clearly is able to explain the period of increased volatility and outperforms also on these grounds the CKLS model. In any case, the GARCH parameters are significant which implies time dependence of the short term interest rate's volatility. In addition, the conditional variance shows a negative intercept whereas GARCH models are only defined for $c_0, c_1, c_2 > 0$. This result depends on the inclusion of dummies as Table 8 shows. This Table also gives the estimates of the traditional GARCH(1,1) model because contrary to the GARCH-X model, its asymptotics are well known. The estimates without any dummies (Table 8) deliver a levels effect parameter γ in the GARCH-X model which is nearly equal to 0.5 as the CIR-SR model predicts but it remains insignificant. Although the GARCH parameters slightly change in comparison to the model with dummies the sum still is smaller than one, and all values are strictly positive. Apart from the implausible negative intercept in the GARCH-X model with dummies, qualitatively it does not matter if dummies are included. In both cases the GARCH parameters are significant whereas the levels effect parameter is not. In the GARCH model, however, the ARCH and GARCH parameters sum up to more than one which violates the definition. Therefore we re-estimate the model with a dummy in the conditional variance equation which is equal to one on February 27, 1981 and March 6, 1981. Now, $c_1 + c_2$ is strictly less than one, as required (see Table 9).

An extension is the EGARCH model. On the one hand it incorporates the leverage effect (unexpected interest rate hikes typically are followed by an increased conditional variance) and on the other hand it ensures positive values for the conditional variance. Table 10 gives the results for the EGARCH-X model as well as for the traditional EGARCH model. Also these estimates reveal that the asymptotic characteristics of the estimators in conditional variance models with levels and ARCH effects is quite problematic. γ reaches an implausibly large (but insignificant) value. Nevertheless, it is to be concluded that leverage and levels effect are not significant in the model that includes both whereas the traditional EGARCH model delivers a positive and significant estimate for the leverage effect parameter, as expected.

Table 7: Estimates of the GARCH-X Model with Dummies

The model estimated with weekly data of the DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \delta_1 D_{2/81} + \delta_2 D_{3/81} + \beta r_{t-1} + u_t$$

$$E[u_t|F_{t-1}] = 0,\ E[u_t^2|F_{t-1}] = h_t,$$

$$h_t = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1} + (c_3 + \delta_3 D_t) r_{t-1}^{2(\gamma+\delta_4 D_t)}$$

$$D_t = [1 \text{ for } t \in (5.10.1979 \text{ - } 24.9.1982),\ 0 \text{ other}]$$

$$D_{2/81} = [1 \text{ for } t = \ 27.2.1981,\ 0 \text{ other}]$$

$$D_{3/81} = [1 \text{ for } t = \ 6.3.1981,\ 0 \text{ other}].$$

	Model with outliers		Model without outliers	
α[a]	0.0177	(0.646)	0.0195	(0.672)
β	-0.0037	(-0.778)	-0.0041	(-0.850)
δ_1			2.9398	(0.676)
δ_2			0.5809	(1.049)
c_0	-0.3025	(-0.784)	-0.1716	(-0.850)
c_1	0.3818	(2.544)	0.3478	(2.019)
c_2	0.4914	(2.770)	0.6454	(2.985)
c_3	0.3296	(0.871)	0.1893	(0.452)
δ_3	-0.0685	(-0.525)	-0.0235	(-0.194)
γ	0.0326	(0.843)	0.0380	(0.493)
δ_4	0.0775	(2.526)	0.0517	(0.427)
HQ[b]	-1.968		-2.022	
RB-LM(1) test[c]	0.712	(0.399)	0.332	(0.565)
RB-LM(11) test	11.56	(0.398)	12.03	(0.361)

[a] t-values are in brackets.

[b] Hannan-Quinn information criterion.

[c] Marginal significance levels are in brackets.

Table 8: Estimates of GARCH-X and GARCH Model

The model estimated with weekly data of the DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t$$

$$E[u_t|F_{t-1}] = 0,\ E[u_t^2|F_{t-1}] = h_t,$$

$$h_t = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1} + c_3 r_{t-1}^{2\gamma}.$$

	GARCH-X-Model		GARCH-Model	
α[a]	0.0203	(0.748)	0.0212	(0.028)
β	-0.0042	(-0.894)	-0.0045	(-1.020)
c_0	0.0327	(0.501)	0.0476	(2.267)
c_1	0.4631	(3.087)	0.4493	(2.963)
c_2	0.4754	(2.637)	0.6286	(4.454)
c_3	0.0045	(0.197)		
γ	0.5961	(0.626)		
HQ[b]	-1.989		-1.823	
RB-LM(1) test[c]	0.972	(0.324)	0.999	(0.318)
RB-LM(11) test	11.29	(0.419)	11.11	(0.434)

[a] t-values are in brackets.

[b] Hannan-Quinn information criterion.

[c] Marginal significance levels are in brackets.

Table 9: Estimates of GARCH(1,1) Model with Dummy for February/March 1981

The model estimated with weekly data of the DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t$$
$$E[u_t|F_{t-1}] = 0,\ E[u_t^2|F_{t-1}] = h_t,$$
$$h_t = c_0 + \delta D_t + c_1 u_{t-1}^2 + c_2 h_{t-1}$$
$$D_t = [1 \text{ for } t = \text{ Febr. 27/March 6, 1981}, 0 \text{ other}].$$

	GARCH-Model	
α^a	0.0179	(0.649)
β	-0.0036	(-0.793)
c_0	0.0720	(2.032)
δ	1.1407	(1.560)
c_1	0.3175	(2.037)
c_2	0.4980	(2.213)
HQ[b]	-1.782	
RB-LM(1) test[c]	0.707	(0.400)
RB-LM(11) test	10.57	(0.480)

[a] t-values are in brackets.

[b] Hannan-Quinn information criterion.

[c] Marginal significance levels are in brackets.

Table 10: Estimates of EGARCH-X and EGARCH Model

The model estimated with weekly data of the DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t,\ u_t = \eta_t \sqrt{h_t},\ \eta_t \sim i.i.d.\, N(0,1)$$
$$\ln(h_t) = \omega_0 + \omega_1 g(\eta_{t-1}) + \omega_2(\ln(h_{t-1})) + \omega_3 r_{t-1}^{2\gamma}$$
$$g(\eta_t) = \Theta\eta_t + \vartheta[|\eta_t| - E[\eta_t]]$$

	EGARCH-X Model		EGARCH Model	
α[a]	0.0194	(0.744)	0.0375	(1.387)
β	-0.0030	(-0.680)	-0.0066	(-1.583)
ω_0	0.0218	(1.497)	0.0160	(1.734)
ω_1	0.2980	(5.254)	0.2756	(6.087)
ω_2	0.6260	(5.365)	0.7447	(8.568)
Θ	0.3791	(1.909)	0.4000	(2.208)
ϑ	0.9511	(4.071)	0.9186	(5.814)
ω_3	0.0001	(0.144)		
γ	1.3390	(0.932)		
$\omega_1\Theta$[b]	0.1130	(1.893)	0.1103	(2.180)
$\omega_1\vartheta$[c]	0.2834	(3.560)	0.2532	(3.020)
HQ[d]	0.1965		0.1826	
RB-LM(1) test[e]	0.005	(0.941)	0.004	(0.949)
RB-LM(11) test	6.802	(0.815)	6.857	(0.811)

[a] t-values are in brackets.

[b] $\omega_1\Theta$ denotes the leverage effect. Its variance is computed as $\mathrm{Var}(\omega_1\Theta) = \Theta^2\,\mathrm{Var}(\omega_1) + \omega_1^2\,\mathrm{Var}(\Theta) + 2\,\Theta\omega_1\,\mathrm{Cov}(\omega_1, \Theta)$.

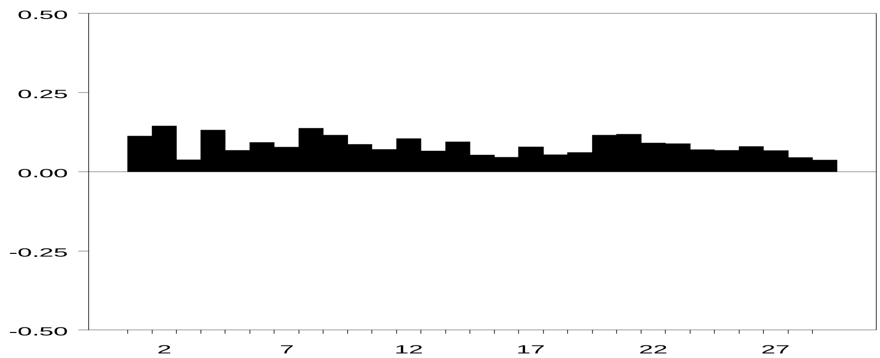
[c] $\omega_1\vartheta$ denotes the ARCH effect parameter. Its variance is computed accordingly.

[d] Hannan-Quinn information criterion.

[e] Marginal significance levels are in brackets.

The conclusion to be drawn thus far is that a model with ARCH and levels effect is overparametrised with respect to the DM 3-Month rate. Within the traditional GARCH and EGARCH models no structural break is detected. The autocorrelation function of the absolute standardised residuals of the GARCH(1,1) model which assumes the mean to be generated by a random walk as in (17) is given in Figure 4. Also on these counts does the model outperform the CKLS model, although many autocorrelation coefficients are significant. The EGARCH models deliver comparable patterns (not given).

Figure 4: Absolute GARCH Residuals Autocorrelation Function



Due to the significant asymmetry parameter in the EGARCH model the GARCH model appears to be misspecified. However, applying the formula given in DROST/WERKER (1996) the latter can easily be translated into a linear two factor term structure model whereas the former would demand auxiliary simulations.

DROST/WERKER (1996) derive a continuous time model which is equivalent to a GARCH model in discrete time.[14] Accordingly the model

$$\begin{aligned} r_t - r_{t-1} &= u_t \qquad u_t \sim i.i.d.\, N(0, h_t) \\ h_t &= c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1} \end{aligned} \tag{17}$$

[14] DROST/WERKER (1996) define a so called *weak* GARCH discrete time model which is closed under time aggregation. Its defintion of the unconditional variance differs from the traditional GARCH model.

with $c_0, c_1, c_2 > 0$, $c_1 + c_2 < 1$ and a finite fourth moment can directly be translated into a continuous time model of the form

$$\begin{aligned} dr_t &= \sigma_{t-}\, dz_1 \\ d\sigma_t^2 &= \kappa(\theta - \sigma_{t-}^2)dt + \sqrt{2\lambda\kappa}\, \sigma_{t-}^2\, dz_2 \end{aligned} \tag{18}$$

where z_1 and z_2 are independent Brownian motions (i.e. $E[dz_1] = 0,\ E[dz_2] = 0,\ E[dz_1 dz_2] = 0$) and with its parameters being determined by c_0, c_1 and c_2 (the distance between two observations $\triangle$ is assumed to approach zero):

$$\frac{c_0/\triangle}{1 - c_1 - c_2} \to \theta \qquad \frac{1 - c_1 - c_2}{\triangle} \to \kappa \qquad \frac{c_1^2}{1 - c_1 - c_2} \to \lambda$$

The discrete time estimates are[15]

$$\begin{aligned} r_t - r_{t-1} &= u_t \qquad u_t \sim i.i.d.\, N(0, h_t) \\ h_t &= 0.0727 + 0.3180 u_{t-1}^2 + 0.4947 h_{t-1}. \end{aligned} \tag{19}$$

With $\triangle$ set to 1 we get the following short term interest rate dynamics

$$\begin{aligned} dr_t &= \sigma_{t-}\, dz_1 \\ d\sigma_t^2 &= 0.19(0.39 - \sigma_{t-}^2)dt + 0.20\, \sigma_{t-}^2\, dz_2 \end{aligned} \tag{20}$$

As shown in e.g. Cox/Ingersoll/Ross (1985) or generally for exponentially affine term structure models in Duffie/Kan (1996) such stochastic differential equations lead to second order partial differential equations for zero coupon bond prices. Consequently, the factor dynamics of (20) determine the entire zero coupon term structure.

A prerequisite for the two-factor model in (20) to explain the stochastics of all interest rates is that a multivariate vector error correction analysis delivers one stochastic trend, i.e. all interest rates need to be cointegrated with r_t since the other factor is stationary by definition. With

[15] The model additionally incorporates a dummy variable for the extreme interest rate values in February and March 1981 as demonstrated in Table 9 because these values are due to institutional irregularities (see also footnote 12).

respect to U.S. data, Johnson (1994), Engsted/Tanggaard (1994), Hall/Anderson/Granger (1992) and Pagan/Hall/Martin (1995) do indeed find one stochastic trend whereas Wolters (1998) finds two stochastic trends for German yields. However, if our second (stationary) factor has an higher influence on short term interest rates in comparison to long term ones this model can explain the relatively high volatility of short term interest rates in comparison to long term ones.[16]

4 Summary and Conclusions

We presented a testing procedure for the restrictions of alternative zero curve arbitrage models which does not lead to invalid distributions of the test statistic. It was shown that within a framework of linear parametric models the data generating process of the Euro-DM 3-Month rate does not exhibit mean reversion. The simplification of the zero curve arbitrage models to assume an AR(1) process cannot be rejected by the RB-LM test. In contrast to previous studies for U.S. data, the volatility depends on either information shocks or the interest rate level but not on both. However, the GARCH model outperforms the levels effect model. Finally, we propose a two factor model of the term structure in Germany, where one factor is the short term interest rate level and the second its conditional variance.

[16] Pfann/Schotman/Tschernig (1996) propose a non-linear two regime model in order to explain this phenomenon as well as mean reversion for double digit interest rate values.

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